

The patching of critical points using quantum group

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Abstract

Following our recent conjecture to model the phenomena of antiferromagnetism and superconductivity by quantum symmetry groups, we discuss in the present note how to construct a workable scenario using this symmetry. In particular we propose to patch the relevant critical points. This means we identify fixed points, corresponding to various k or q [since the two are related] and make expansion around these points, to control these expansion we can impose gauge structure and we thus arrive at quantum group based gauge theory or collection of classical gauge theories which represent the condensed matter system such as cuprates. This is different than ordinary gauge theories in several ways, for in ordinary field theory one has well-defined critical point here the critical points are not unique or simple. In short the real transition than in condensed matter system is represented by collection or *ensemble average* of the several chosen critical points which come from ordinary field theory [gauge theory]. This idea may reveal the connection between Hubbard model and gauge theory and string theory. In short it can lead to the non-perturbative formulation of Hubbard and other condensed matter Hamiltonians.

In a previous work one of us [1] have advanced the conjecture that one should attempt to model the phenomena of antiferromagnetism and superconductivity by using quantum symmetry group. Following this conjecture to model the phenomena of antiferromagnetism and superconductivity by quantum symmetry groups, three toy models were proposed [2], namely, one based on $SO_q(3)$ the other two constructed with the $SO_q(4)$ and $SO_q(5)$ quantum groups. Possible motivations and rationale for these choices are were outlined. In [3] a model to describe quantum liquids in transition from 1d to 2d dimensional crossover using quantum groups was outlined.

In this short note we turn our attention to an idea to construct a theory based on patching critical points so as to simulate the behavior of systems such as cuprates. To illustrate our idea we start with an example which has been considered by Frahm et al., [4]. The model deals with antiferromagnetic spin-1 chain doped with spin-1/2 carriers. One can write the Hamiltonian as consisting of two parts exchange and hopping

$$\begin{aligned}
H &= \sum_{n=1}^L H_{n,n+1}^{exch} + H_{n,n+1}^{hopp}, \\
H_{i,j}^{exch} &= \frac{1}{2} \left(\frac{1}{S_i S_j} \mathbf{S}_i \cdot \mathbf{S}_j - 1 + \delta_{S_i S_j, 1} [1 - (\mathbf{S}_i \cdot \mathbf{S}_j)^2] \right), \\
H_{i,j}^{hopp} &= -(1 - \delta_{S_i S_j, 1}) P_{ij} (\mathbf{S}_i \cdot \mathbf{S}_j)
\end{aligned} \tag{1}$$

here as usual the quantity $\mathbf{S}_i^2 = S_i(S_i + 1)$ with S_i taking the value 1 or 1/2 and P_{ij} permutes the spins on sites i and j .

This Hamiltonian is different from the Hamiltonian generally thought to describe the carrier doped Haldane system $Y_{2-x}Ca_xBaNiO_5$, namely

$$\begin{aligned}
H &= \sum_{n=1}^L H_{n,n+1}^{exch} + H_{n,n+1}^{hopp}, \\
H_{i,j}^{exch} &= J \delta_{S_i S_j, 1} \mathbf{S}_i \cdot \mathbf{S}_{j+1}, \\
H_{i,j}^{hopp} &= -P_{ij} (\mathbf{S}_i \cdot \mathbf{S}_j + 1/2).
\end{aligned} \tag{2}$$

For no doping $x = 0$ the interactions between the spin-1 Ni^{+} reduce to the usual case of Heisenberg model, however upon doping one gets a low energy doublet state in an effective one-band Hamiltonian which can move in the $S=1$ background which is caused by the mixing of spin $S=1/2$ holes on the oxygen sites. The main difference between the two Hamiltonians is the biquadratic term. Eq. 1 which contains such a term gives spin-1 Takhtajan-Babujian chain for hole doping $x = 0$, thus in this undoped limit the spectrum would be expected to be gapless, however it is claimed in [4] that it is possible to reintroduce the gap in the continuum limit where there is a field theoretical description of the model. This theoretical description is effective field theory, as should be noted with the following observation which are of interest:

- It is known that in the undoped limit [$x = 0$] one obtains the $SU(2)_{k=2}$ [which is related to quantum group [1,2,5,6]] Wess-Zumino-Witten (WZW) * with central charge $c = 3/2$. It is readily seen that this model is equivalent to three massless Majorana fermions which are a triplet under $SU(2)$.
- For the other extreme case $x = 1$ [the filled band] of the $S = 1$ chain one obtains $SU(2)_{k=1}$ in the low energy field theory limit.
- The mixed case or finite doping case one obtains one free bosonic mode in the charge sector, the spin sector contains a direct sum of the $c = 3/2$ and $c = 1/2$ models with different velocities! thus by doping a fourth Majorana fermion is generated and this feature is observed in two-channel Kondo physics

Keeping these points we look at the above in the following manner:-

*In literature WZW is also called Wess-Zumino-Novikov-Witten [WZNW] to be fair to Novikov!

- The various $SU(2)_k$ corresponds to infrared stable fixed points [5,6]. Thus the undoped and the fully doped case correspond to two different fixed points. The doped case is an admixture of the two as shown in [4] and interpolates between the spin $S=1$ and $S=1/2$ states. In a like manner we propose to first label the fixed points by choosing a symmetry such as $SO(N)_q$, or $SU(N)_q$ [or other Lie Groups of ones choice to describe the condensed matter system] which correspond to certian k level of WZW model.
- In the next step since these k values correspond to particular WZW models we identify their low energy spectra.
- From the low energy spectra we can go to a full gauge theory structure.
- We can next define a ‘partition’ function of the critical points which weighs the contributions of the degrees of freedom which correspond to the chosen critical points.

In this sense we can construct a representative effective or smeared gauge structure for the condensed matter system in other words a quantum group based gauge structure. One can look at this as an expansion about the critical points of the relevant degrees of freedom using quantum groups. We note that a strong feature of quantum groups is that they unify classical Lie algebras and topology. In general sense it is expected that quantum groups will lead to a deeper understanding of the concept of symmetry in condensed matter physics.

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